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Delocalized, non-SUSY p-branes, tachyon condensation and tachyon matter

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Abstract: We construct non-supersymmetric p-brane solutions of type II supergravities in arbitrary dimensions (d) delocalized in one of the spatial transverse directions. By a Wick rotation we convert these solutions into Euclidean p-branes delocalized in the transverse time-like direction. The former solutions in $d = 10$ nicely interpolate between the $(p + 1)$-dimensional non-BPS D-branes and the p-dimensional BPS D-branes very similar to the picture of tachyon condensation for the tachyonic kink solution on the non-BPS D-branes. On the other hand the latter solutions interpolate between the $(p+1)$-dimensional non-BPS D-branes and the tachyon matter supergravity configuration very similar to the picture of rolling tachyon on the non-BPS D-branes.

Keywords: Superstrings and Heterotic Strings, p-branes, D-branes, Tachyon Condensation.

Contents

1. Introduction

In [1] we constructed static, non-supersymmetric p-brane solutions of type II supergravities in d-dimensions and showed how they interpolate between chargeless p-brane–anti p-brane solutions and BPS p-brane solutions. So, the two different brane solutions of the two sides of the interpolation have the same dimensionalities. However, for the case of non-BPS branes, the tachyon condensation on the kink solution reduces the non-BPS D$(p+1)$-branes to codimension one BPS Dp-branes [2, 3]. Therefore, the two brane solutions in this case have dimensionalities differing by one. The most natural way to see this picture emerging from a supergravity solution (in the absence of explicit appearance of the tachyon field) is to consider the non-supersymmetric p-brane solutions delocalized in one of the transverse spatial directions. The purpose of this paper is to construct such solutions and study their properties, in particular, we will try to understand how the BPS Dp-branes arise from the non-BPS D$(p+1)$-branes [3] and also how the supergravity configuration of tachyon matter [4]-[6] arise from these solutions.

For BPS D-branes, the difference in dimensionalities (i.e. Dp $\to$ D$(p+1)$, or D$(p+1)$ $\to$ Dp) appear due to T-duality transformation and in this process the theory also changes from type IIA (IIB) to type IIB (IIA). For example, to construct a D$(p+1)$-brane from a Dp-brane, one first delocalizes the Dp-brane solution in type IIA (or IIB) theory by placing a continuous array of Dp-branes along one of the transverse spatial directions (the T-dual direction). This produces an isometry in that particular direction and then the application of T-duality along this direction produces a localized D$(p+1)$-brane solution in type IIB (or IIA) theory [7]. This procedure works because the BPS branes do not interact with each other. However, because the non-supersymmetric branes interact, it is not clear how the above process of delocalization will work. This is the reason we have to explicitly solve the equations of motion of type II supergravities containing a metric, a dilaton and a $q = d - p - 2$ form field-strength.[1] We use a specific ansatz for the metric and the form-field to solve the equations of motion and obtain delocalized, non-supersymmetric p-branes

[1] Similar delocalized solutions were also constructed in a different context in an earlier work in [8].

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characterized by four independent parameters. We show that, unlike the BPS p-branes, it is possible to convert these solutions to fully localized $(p+1)$-branes, without taking T-duality, if the parameters satisfy certain condition. We recognize these to be the non-BPS D$(p+1)$-brane solutions [9, 10, 1] of the same theory as the original p-brane solutions. This also explains why we have non-BPS D-branes of odd and even dimensionalities in type IIA and type IIB string theories respectively [2]. By scaling certain parameters of the delocalized solutions appropriately, we show how these solutions reduce to BPS Dp-brane solutions. We therefore interpret these solutions as the interpolating solutions between non-BPS D$(p+1)$-branes and the BPS Dp-branes very similar to the tachyon condensation for the tachyonic kink solution on the non-BPS D$(p+1)$-branes [2, 3].

Next we Wick rotate these solutions which simply exchanges the delocalized spatial direction with the time-like direction of the non-supersymmetric p-branes. We therefore end up getting Euclidean p-branes delocalized along the transverse time-like direction of the branes. We find that these solutions will be real only if they do not possess any charge and so, they are characterized by three parameters. As discussed before, these solutions can also be converted to fully localized $(p+1)$-branes i.e. non-BPS D$(p+1)$-branes if the parameters satisfy certain condition. However, since in this case one can not have charged solutions, we find that it is not possible to obtain completely localized Euclidean p-branes (or S-branes) [11]–[15] from the delocalized solutions by scaling parameters as was mentioned for the spatially delocalized case. On the other hand, we show that by adjusting the parameters it is possible to obtain the supergravity configuration of tachyon matter [6] from these Wick rotated solutions. We therefore interpret these solutions as the interpolating solutions between non-BPS D$(p+1)$-branes and the tachyon matter very similar to the rolling tachyon solution [16] of the non-BPS D$(p+1)$-branes discussed by Sen.

This paper is organized as follows. In section 2, we construct and discuss the properties of spatially delocalized, non-supersymmetric p-branes. The Wick rotated versions and their properties are discussed in section 3. We conclude in section 4.

2. Spatially delocalized, non-SUSY p-branes

In this section we construct and study the properties of the non-supersymmetric p-branes in d-dimensions delocalized along one of the $(d-p-1)$ spatial transverse directions. The relevant supergravity action in the Einstein frame has the form,

$$S = \int d^d x \sqrt{-g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2 \cdot q!} e^{a\phi} F_{[q]}^2 \right], \tag{2.1}$$

where $g_{\mu\nu}$, with μ, $\nu = 0, 1, \ldots, d-1$ is the metric and $g = \det(g_{\mu\nu})$, R is the scalar curvature, ϕ is the dilaton, $F_{[q]}$ is the field strength of a $(q-1) = (d-p-3)$-form gauge field and a is the dilaton coupling.

We will solve the equations of motion following from (2.1) with the ansatz for the metric and the q-form field strength as given below,

$$\begin{aligned} ds^2 &= e^{2A(r)} \left(dr^2 + r^2 d\Omega_{d-p-3}^2 \right) + e^{2B(r)} \left(-dt^2 + dx_1^2 + \cdots + dx_p^2 \right) + e^{2C(r)} dx_{p+1}^2 \\ F_{[q]} &= b \, \mathrm{Vol}(\Omega_{d-p-3}) \wedge dx_{p+1} \, . \end{aligned} \tag{2.2}$$

In the above $r = (x_{p+2}^2 + \cdots + x_{d-1}^2)^{1/2}$, $d\Omega_{d-p-3}^2$ is the line element of a unit $(d-p-3)$-dimensional sphere, $\mathrm{Vol}(\Omega_{d-p-3})$ is its volume-form and b is the magnetic charge parameter. The solutions (2.2) represent magnetically charged p-branes delocalized in a transverse spatial direction x_{p+1}. It should be clear from the form of r given before which says that the true transverse directions are $x_{p+2}, \ldots, x_{d-1}$. On the other hand x_{p+1} is neither a transverse direction nor a brane direction (since $B(r) \neq C(r)$ in general), but represents the delocalized direction. We will also use a gauge condition

$$(p+1)B(r) + (q-2)A(r) + C(r) = \ln G(r)\,. \tag{2.3}$$

Note that when $G(r) = 1$, the above condition reduces to the extremality or supersymmetry condition [17]. We call $G(r)$ as the non-extremality function, whose extremal limit is $G(r) \to 1$.

Using (2.2) and (2.3) the various components of Einstein equation and the dilaton equation take the forms,

$$B'' + \frac{q-1}{r}B' + \frac{G'}{G}B' - \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2(q-1)}} = 0 \tag{2.4}$$

$$C'' + \frac{q-1}{r}C' + \frac{G'}{G}C' + \frac{b^2(p+1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2(q-1)}} = 0 \tag{2.5}$$

$$A'' + \frac{q-1}{r}A' + \frac{G'}{G}\left(A' + \frac{1}{r}\right) + \frac{b^2(p+1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2(q-1)}} = 0 \tag{2.6}$$

$$\begin{aligned} -A'' - \frac{G''}{G} + \frac{G'^2}{G^2} - \frac{1}{p+1}\left(\frac{G'}{G} - (q-2)A' - C'\right)^2 - (q-2)A'^2 + \\ + \frac{G'}{G}A' - \frac{q-1}{r}A' - C'^2 - \frac{1}{2}\phi'^2 + \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2(q-1)}} = 0 \end{aligned} \tag{2.7}$$

$$\phi'' + \frac{q-1}{r}\phi' + \frac{G'}{G}\phi' - \frac{ab^2}{2}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2(q-1)}} = 0\,. \tag{2.8}$$

In the above 'prime' denotes derivative with respect to r. Using (2.3), (2.4) and (2.5) in eq. (2.6) we obtain an equation for the non-extremality function as,

$$\frac{G''}{G} + \frac{(2q-3)}{r}\frac{G'}{G} = 0 \tag{2.9}$$

There are three different solutions to this equation and they are

$$(i)\ \ G = 1 - \frac{\omega^{2(q-2)}}{r^{2(q-2)}}\,, \qquad (ii)\ \ G = 1 + \frac{\omega^{2(q-2)}}{r^{2(q-2)}}\,, \qquad (iii)\ \ G = \frac{\omega^{2(q-2)}}{r^{2(q-2)}}\,. \tag{2.10}$$

The solution in (iii) can be shown to be supersymmetric by a coordinate transformation and lead to the near horizon limits of delocalized BPS p-brane solutions [1]. Since we are interested in non-supersymmetric solutions we do not consider (iii). Also the solution (ii) is not of our interest since it gives non-supersymmetric delocalized p-brane solutions which have BPS limits leading to some unusual brane configuration and not the usual BPS p-brane configuration [1]. Since we are interested in interpolating solutions between non-BPS D$(p+1)$-branes and the usual BPS Dp-branes we will consider only case (i). The non-extremality function in this case can be factorized as follows,

$$G(r) = \left(1 + \frac{\omega^{q-2}}{r^{q-2}}\right)\left(1 - \frac{\omega^{q-2}}{r^{q-2}}\right) = H(r)\tilde{H}(r)\,, \tag{2.11}$$

where $H(r) = 1 + \omega^{q-2}/r^{q-2}$, $\tilde{H}(r) = 1 - \omega^{q-2}/r^{q-2}$, with ω^{q-2}, a real parameter. Now from (2.4) and (2.8) we express ϕ in terms of B and also from (2.4) and (2.5) we express C in terms of B as follows,

$$\phi = \frac{a(d-2)}{q-1}B + \delta_1 \ln\frac{H}{\tilde{H}} \tag{2.12}$$

$$C = -\frac{p+1}{q-1}B + \delta_2 \ln\frac{H}{\tilde{H}} \tag{2.13}$$

where δ_1 and δ_2 are two real and negative integration constants which can be understood if one actually finds the above solutions from the corresponding equations of motion. We can also determine A in terms of B using (2.3) and (2.13) as,

$$A = -\frac{p+1}{q-1}B - \frac{\delta_2}{q-2}\ln\frac{H}{\tilde{H}} + \frac{1}{q-2}\ln(H\tilde{H})\,. \tag{2.14}$$

We therefore have to solve B from eq. (2.4) and check whether the solution is consistent with eq. (2.7). In order to solve B we make an ansatz

$$e^B = F^\gamma$$

with,

$$F = \cosh^2\theta\left(\frac{H}{\tilde{H}}\right)^\alpha - \sinh^2\theta\left(\frac{\tilde{H}}{H}\right)^\beta\,, \tag{2.15}$$

where α, β, θ and γ are real constants and we will comment on them later. Substituting (2.15) in eq. (2.4) we find that the solutions exist provided the parameters satisfy the following relations,

$$\gamma\chi = -2, \quad \alpha - \beta = a\delta_1\,, \qquad b = \sqrt{\frac{4(d-2)}{\chi(q-1)}}(q-2)(\alpha+\beta)\omega^{q-2}\sinh 2\theta\,, \tag{2.16}$$

where $\chi = 2(p+1) + a^2(d-2)/(q-1)$. Note that we have taken $b \geq 0$ and kept both sign choices for $\alpha+\beta$ for later convenience. The sign of $\alpha+\beta$ determines the sign for θ, given $b > 0$ in (2.16). These solutions are consistent with eq. (2.7) provided the parameters satisfy

$$\frac{1}{2}\delta_1^2 + \frac{2\alpha(\alpha - a\delta_1)(d-2)}{\chi(q-1)} = (1-\delta_2^2)\frac{q-1}{q-2}\,. \tag{2.17}$$

From (2.17) and $\alpha - \beta = a\delta_1$, we can express α and β in terms of δ_1 and δ_2 as,

$$\begin{aligned}\alpha &= \pm\sqrt{\frac{\chi(q-1)^2}{2(d-2)(q-2)}(1-\delta_2^2) - \frac{\delta_1^2}{2}\frac{(q-1)(p+1)}{(d-2)}} + \frac{a\delta_1}{2}\\ \beta &= \pm\sqrt{\frac{\chi(q-1)^2}{2(d-2)(q-2)}(1-\delta_2^2) - \frac{\delta_1^2}{2}\frac{(q-1)(p+1)}{(d-2)}} - \frac{a\delta_1}{2}\,.\end{aligned} \tag{2.18}$$

We thus find from the above relations that both δ_2 and δ_1 are bounded by[2]

$$\begin{aligned} |\delta_2| &\leq 1 \\ |\delta_1| &\leq \sqrt{\frac{\chi(q-1)(1-\delta_2^2)}{(q-2)(p+1)}}\,. \end{aligned} \tag{2.19}$$

Since we found $\gamma = -2/\chi$, we obtain from (2.12)–(2.15)

$$\begin{aligned} e^{2A} &= F^{\frac{4(p+1)}{(q-1)\chi}} (H\tilde{H})^{\frac{2}{q-2}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{2\delta_2}{q-2}} \\ e^{2B} &= F^{-\frac{4}{\chi}} \\ e^{2C} &= F^{\frac{4(p+1)}{(q-1)\chi}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_2} \\ e^{2\phi} &= F^{-\frac{4a(d-2)}{(q-1)\chi}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_1}\,. \end{aligned} \tag{2.20}$$

So, the complete non-supersymmetric p-brane solutions delocalized in transverse x_{p+1} direction have the forms,

$$\begin{aligned} ds^2 &= F^{\frac{4(p+1)}{(q-1)\chi}}(H\tilde{H})^{\frac{2}{q-2}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{2\delta_2}{q-2}}\!\left(dr^2+r^2d\Omega^2_{d-p-3}\right)+F^{-\frac{4}{\chi}}\left(-dt^2+\sum_{i=1}^{p}dx_i^2\right)+F^{\frac{4(p+1)}{(q-1)\chi}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_2}dx^2_{p+1} \\ e^{2\phi} &= F^{-\frac{4a(d-2)}{(q-1)\chi}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[q]} = b\,\mathrm{Vol}(\Omega_{d-p-3}) \wedge dx_{p+1}\,. \end{aligned} \tag{2.21}$$

Note that unlike the localized solutions [18, 1], which are characterized by three parameters, the delocalized solutions are charaterized by four parameters ω, θ, δ_1 and δ_2 (α and β are given in terms of δ_1 and δ_2 as in (2.18) and b is related to δ_1, δ_2, ω and θ by (2.16)). We thus find that the delocalization actually introduces one more parameter in the non-supersymmetric solutions and this does not happen for BPS solutions. This will prove crucial to interpret these solutions as interpolating solutions between non-BPS D$(p+1)$ branes and BPS Dp-branes. Also since these solutions are non-supersymmetric, the four parameters would presumably be related to the mass, the charge, the tachyon vev $\langle T\rangle$ and the vev of its derivative $\langle \partial_x T\rangle$ of the non-supersymmetric p-branes. However, the microscopic string interpretation of these solutions and also the precise relationships of these parameters and the physical parameters just mentioned are not clear to us.

In $d = 10$, the delocalized p-brane solutions (2.21) take the forms,

$$\begin{aligned} ds^2 &= F^{\frac{p+1}{8}}(H\tilde{H})^{\frac{2}{6-p}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{2\delta_2}{6-p}}\left(dr^2+r^2d\Omega^2_{7-p}\right)+F^{-\frac{7-p}{8}}\left(-dt^2+\sum_{i=1}^{p}dx_i^2\right)+ \\ &\quad + F^{\frac{p+1}{8}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_2}dx^2_{p+1} \\ e^{2\phi} &= F^{-a}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[q]} = b\,\mathrm{Vol}(\Omega_{7-p})\wedge dx_{p+1} \end{aligned} \tag{2.22}$$

[2]The solutions also exist when these bounds are violated [1] but their BPS limits do not give the usual BPS p-branes, therefore they, as mentioned earlier, are not considered in this paper.

where F is as given in (2.15) and $H = 1 + \omega^{6-p}/r^{6-p}$, $\tilde{H} = 1 - \omega^{6-p}/r^{6-p}$ and also $\chi = 32/(7-p)$.

Once we know the form of the metric, we can calculate the energy-momentum (e-m) tensor associated with the brane from the linearized form of Einstein equation given by,

$$\nabla^2 \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \right) = -2\kappa_0^2 T_{\mu\nu} \delta^{(8-p)}(r)\,, \tag{2.23}$$

where we have expanded the metric around asymptotically flat space as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and used the harmonic gauge $\partial_\lambda h^\lambda_\mu - \frac{1}{2}\partial_\mu h = 0$ with $h = \eta^{\mu\nu} h_{\mu\nu}$. Also in (2.23) $2\kappa_0^2 = 16\pi G_{10}$, G_{10} being the ten dimensional Newton's constant. From (2.22) we find

$$\begin{aligned}
h_{00} &= \frac{7-p}{8}\left[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta)\right]\frac{\omega^{6-p}}{r^{6-p}}\\
h_{ij} &= -\frac{7-p}{8}\left[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta)\right]\frac{\omega^{6-p}}{r^{6-p}}\delta_{ij}\\
h_{xx} &= \left\{\frac{p+1}{8}\left[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta)\right] + 4\delta_2\right\}\frac{\omega^{6-p}}{r^{6-p}}\\
h_{mn} &= \left\{\frac{p+1}{8}\left[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta)\right] - \frac{4\delta_2}{6-p}\right\}\frac{\omega^{6-p}}{r^{6-p}}\delta_{mn}\\
h &= \left\{\frac{p+1}{4}\left[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta)\right] - \frac{8\delta_2}{6-p}\right\}\frac{\omega^{6-p}}{r^{6-p}}\,,
\end{aligned} \tag{2.24}$$

where $i,j = 1,\ldots,p$, $x = x_{p+1}$ and $m,n = p+2,\ldots,9$. Substituting (2.24) in (2.23) we obtain,

$$\begin{aligned}
T_{00} &= \frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)\omega^{6-p}\left[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta) - \frac{4\delta_2}{6-p}\right]\\
T_{ij} &= -\frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)\omega^{6-p}\left[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta) - \frac{4\delta_2}{6-p}\right]\delta_{ij}\\
T_{xx} &= \frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)\omega^{6-p}\left[\frac{4\delta_2(7-p)}{6-p}\right]\\
T_{mm} &= 0\,.
\end{aligned} \tag{2.25}$$

Here $\Omega_n = 2\pi^{(n+1)/2}/\Gamma((n+1)/2)$ is the volume of the n-dimensional unit sphere. In the above T_{00} is nothing but the ADM mass of the brane. It has the dimensionality mass per unit $(p+1)$-brane volume and therefore shows that the energy is spread also along the delocalized direction $x = x_{p+1}$ as expected. The fact that the brane is spread along x can also be seen from T_{xx} in (2.25) which is non-vanishing. $T_{mm} = 0$ implies that the brane is localized along $m = x_{p+2},\ldots,x_{d-1}$ directions and they are the true transverse directions.

Now let us look at the metric in (2.22). These represent non-supersymmetric p-branes delocalized in x_{p+1} direction in $d = 10$. Note that for BPS case one can make such solutions localized $(p+1)$-brane by a T-duality transformation and so if the p-brane is a solution to type IIA (or IIB) theory then $(p+1)$-brane is a solution of type IIB (or IIA) theory. However, in this case it is possible to make the p-brane solution to a localized $(p+1)$-brane without taking T-duality by simply putting $\theta = 0$ and $2\delta_2 = -\alpha$ (note that this is possible because of the presence of the extra parameters which are not present for the BPS solutions. Here we choose a plus sign in (2.18).). To make it clear note from the last two terms of the metric in (2.22) that for the coefficients of these two terms to match (which is necessary to make it a metric for localized $(p+1)$-brane) F must be some powers of $(H/\tilde{H})$ and from (2.15) we find that this happens only for $\theta = 0$. The coefficients would then match for $\alpha = -2\delta_2$. From (2.16) we see that $\theta = 0$ corresponds to $b = 0$ i.e. the solutions must be chargeless. Also from the expressions of the e-m tensors we see that for $\theta = 0$ and $\alpha = -2\delta_2$, $T_{00} = -T_{ii}$ for $i = 1, \ldots, (p+1)$, i.e. we have a localized $(p+1)$-brane. The solutions and the e-m tensors then take the forms,

$$
\begin{aligned}
ds^2 &= (H\tilde{H})^{\frac{2}{6-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\alpha + \frac{\alpha}{6-p}} \left(dr^2 + r^2 d\Omega_{7-p}^2\right) + \left(\frac{H}{\tilde{H}}\right)^{-\frac{7-p}{8}\alpha} \left(-dt^2 + \sum_{i=1}^{p+1} dx_i^2\right) \\
e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-a\alpha + 2\delta_1}, \qquad F_{[q]} = 0 \qquad (2.26) \\
T_{00} &= -T_{ii} = \frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)\omega^{6-p}\left[\frac{2\alpha(7-p)}{6-p}\right]. \qquad (2.27)
\end{aligned}
$$

This is the supergravity configuration of non-BPS D$(p+1)$-brane discussed by Sen [2] and were also obtained in refs. [9, 10, 1]. The solutions in this case are characterized by two parameters ω and α (or δ_2). The parameter relation (2.17) takes the form,

$$
\delta_1^2 + \alpha(\alpha - a\delta_1) = \frac{(4-\alpha^2)(7-p)}{2(6-p)}. \qquad (2.28)
$$

This determines δ_1 in terms of α (or δ_2) and to have real and negative δ_1, we must have

$$
|\alpha| \leq \frac{8}{\sqrt{2(5p + 14 - p^2)}}. \qquad (2.29)
$$

We also point out that for $p =$ even (odd) the original delocalized p-branes in (2.22) represent solutions in type IIA (IIB) string theory. But since we made the solutions to localized $(p+1)$-branes without T-duality transformation then the solutions in (2.26) also represent solutions in the same theory i.e. in type IIA (IIB) theory for $p =$ odd (even). This clarifies the reason why the non-BPS branes (of the type discussed by Sen) in type IIA and IIB string theories have the wrong dimensionalities compared to the BPS branes.

Now in order to see how the delocalized solutions (2.22) reduce to BPS p-branes, we need the necessary condition from the expressions of the e-m tensors in (2.25), $T_{xx} = 0$ and $T_{00} = -T_{ii}$ for $i = 1, \ldots, p$. This condition means we take either $\delta_2 \to 0$ or $\omega \to 0$. Examining the metric (2.22) carefully, we have the correct BPS limit by sending $|\theta| \to \infty$ while having $\omega \to 0$ according to the following

$$
\begin{aligned}
\omega^{6-p} &\to \epsilon\,\bar{\omega}^{6-p} \\
(\alpha + \beta)\sinh 2\theta &\to \epsilon^{-1}, \qquad (2.30)
\end{aligned}
$$

where ϵ is a dimensionless parameter which tends to zero. With the above scaling $b \to (6-p)\bar{\omega}^{6-p}$, $F \to \bar{H} = 1 + \frac{\bar{\omega}^{6-p}}{r^{6-p}}$, and H, $\tilde{H} \to 1$. Since both δ_1 and δ_2 are bounded given in (2.19), it can be easily checked that the configuration (2.22) reduce to

$$\begin{aligned} ds^2 &= \bar{H}^{\frac{p+1}{8}} \left(dr^2 + r^2 d\Omega_{7-p}^2 + dx_{p+1}^2\right) + \bar{H}^{-\frac{7-p}{8}} \left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) \\ e^{2\phi} &= \bar{H}^{-a}, \qquad F_{[q]} = b \mathrm{Vol}(\Omega_{7-p}) \wedge dx_{p+1} \,. \end{aligned} \tag{2.31}$$

This is precisely the BPS Dp-brane solutions delocalized in x_{p+1} direction. Note from (2.25) that even in this case $T_{xx} \to 0$ and $T_{00} = -T_{ii} \to \frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)\bar{\omega}^{6-p}$. However, this delocalization is trivial in the sense that since $T_{xx} = 0$, we can always replace the line source along x-direction by a delta function source without any cost of energy (true for BPS branes). In other words, in calculating the e-m tensor we replace the Poisson's equation of the harmonic function $\bar{H}$ as,

$$\nabla^2 \bar{H} = -\Omega_{7-p}(6-p)\bar{\omega}^{6-p}\delta^{(8-p)}(r) \to \nabla^2 \bar{H} = -\Omega_{8-p}(7-p)\bar{\omega}^{7-p}\delta^{(9-p)}(r) \,. \tag{2.32}$$

The harmonic function now takes the form $\bar{H} = 1 + \bar{\omega}^{7-p}/r^{7-p}$ where r includes $x \equiv x_{p+1}$ The e-m tensor will be given as $T_{00} = -T_{ii} \to \frac{\Omega_{8-p}}{2\kappa_0^2}(7-p)\bar{\omega}^{7-p}$, for $i = 1, \ldots, p$ and (2.30) will reduce to the localized Dp-brane solutions.

This therefore shows how the delocalized, non-supersymmetric p-brane solutions (2.22) can be regarded as the interpolating solutions between the non-BPS D$(p+1)$-branes and BPS Dp-branes very similar to the tachyon condensation on the tachyonic kink solution on the non-BPS branes.

3. Wick rotation and delocalized, non-SUSY p-branes

In this section we will Wick rotate the spatially delocalized solutions (2.22) and obtain temporally delocalized p-branes as follows. Let us make the following Wick rotation,

$$\begin{aligned} x_{p+1} &\to it \\ t &\to ix_{p+1} \,. \end{aligned} \tag{3.1}$$

Since the harmonic functions H and $\tilde{H}$ are independent of both x_{p+1} and t, under the above change the configurations (2.22) become,

$$\begin{aligned} ds^2 &= F^{\frac{p+1}{8}} (H\tilde{H})^{\frac{2}{6-p}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{2\delta_2}{6-p}} \left(dr^2 + r^2 d\Omega_{7-p}^2\right) + F^{-\frac{7-p}{8}} \sum_{i=1}^{p+1} dx_i^2 - F^{\frac{p+1}{8}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_2} dt^2 \\ e^{2\phi} &= F^{-a} \left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[q]} = ib \ \mathrm{Vol}(\Omega_{7-p}) \wedge dt \,. \end{aligned} \tag{3.2}$$

Note that under the Wick rotation (3.1) the field strength has become imaginary and so, if we insist on real solutions b must vanish or in other words, the solutions in this case must be chargeless. $b = 0$ implies $\theta = 0$ by (2.16) and so, F in (2.15) takes the form,

$$F = \left(\frac{H}{\tilde{H}}\right)^{\alpha} \,. \tag{3.3}$$

So, the real solutions in this case become,

$$
\begin{aligned}
ds^2 &= (H\tilde{H})^{\frac{2}{6-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\alpha - \frac{2\delta_2}{6-p}} \left(dr^2 + r^2 d\Omega_{7-p}^2\right) + \left(\frac{H}{\tilde{H}}\right)^{-\frac{7-p}{8}\alpha} \sum_{i=1}^{p+1} dx_i^2 - \\
&\quad - \left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\alpha + 2\delta_2} dt^2 \\
e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-a\alpha + 2\delta_1}, \qquad F_{[q]} = 0\,.
\end{aligned}
\tag{3.4}
$$

Therefore, unlike the case of spatially delocalized solutions, the temporally delocalized solutions are characterized by three parameters ω, δ_1 and δ_2 (α is related to δ_1 and δ_2 by (2.18)). The brane directions in (3.4) are all spatial and so they are Euclidean branes (or S-branes) delocalized in the transverse time-like direction. However, because these solutions are chargeless, it is not possible to obtain the localized S-branes [11]–[15] by localizing the time direction as was done for the case of spatially delocalized solutions.

As before we calculate the various components of the e-m tensor from the metric in (3.4) as,

$$
\begin{aligned}
T_{00} &= -\frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)\omega^{6-p}\left[\frac{4\delta_2(7-p)}{6-p}\right] \\
T_{ij} &= -\frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)\omega^{6-p}\left[2\alpha - \frac{4\delta_2}{6-p}\right]\delta_{ij} \\
T_{mm} &= 0\,,
\end{aligned}
\tag{3.5}
$$

where in the above $i, j = 1, \ldots, p+1$ and $m = p+2, \ldots, 9$. We also note from (3.5) that for $\alpha = -2\delta_2$, $T_{00} = -T_{ii}$ as expected of a localized $(p+1)$-brane. Indeed we find that under this condition, the coefficient of $-dt^2$ and the coefficient of $(dx_1^2 + \cdots + dx_{p+1}^2)$ term match. The configurations (3.4) in this case reduce to

$$
\begin{aligned}
ds^2 &= (H\tilde{H})^{\frac{2}{6-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\alpha + \frac{\alpha}{6-p}} \left(dr^2 + r^2 d\Omega_{7-p}^2\right) + \left(\frac{H}{\tilde{H}}\right)^{-\frac{7-p}{8}\alpha} \left(-dt^2 + \sum_{i=1}^{p+1} dx_i^2\right) \\
e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-a\alpha + 2\delta_1}, \qquad F_{[q]} = 0\,.
\end{aligned}
\tag{3.6}
$$

This is precisely the non-BPS D$(p+1)$-brane solutions obtained before in (2.26), although our starting solutions in these two cases are different.

On the other hand, we note that the time direction can not be made true transverse direction of the brane by adjusting or scaling the parameters as was done for the spatially delocalized solutions. From the e-m tensors in (3.5), however, it might seem that it is possible to achieve that either for $\delta_2 \to 0$ or for $\omega^{6-p} \to 0$, when T_{00} vanishes. (Note that this happens for S-branes where time is the true transverse direction of the Euclidean or S-branes.) But it is clear from the metric in (3.4) that the coefficients of $-dt^2$ term and $(dr^2 + r^2 d\Omega_{7-p}^2)$ term do not match for $\delta_2 = 0$. So, even if T_{00} vanishes, 't' does not become a transverse direction of the brane. This happens because T_{00} encodes only the linear property of the metric. In the other limit $\omega^{6-p} \to 0$, $T_{00} \to 0$, but since α and δ_2 are finite, we also have $T_{ij} \to 0$ and the metric becomes trivial i.e. the flat space. Note that this did not happen for the spatially delocalized solutions because the solutions were charged and involved more parameters which were scaled appropriately to obtain the localized BPS Dp-brane solutions. However, in this case we can keep T_{00} = fixed and send $T_{ij} \to 0$ by allowing $\alpha \to 2\delta_2/(6-p)$ (Note that this is possible only when the minus sign is chosen in (2.18).). This is exactly the configuration one gets for the tachyon dust or tachyon matter [16] which is pressureless and possesses fixed energy. This configuration is possible because of the extra parameter δ_2 in the solutions. Now with the condition

$$\alpha = \frac{2\delta_2}{6-p} \tag{3.7}$$

the solutions (3.4) will be characterized by two parameters only, namely, ω and δ_1 (since δ_1 and δ_2 are related by (2.18)). However, we would like to point out that this is not the end of the story. It is known from the string field theory [20, 21] as well as the tachyon effective action analysis [22], that for the rolling tachyon, when the tachyon condenses, not only the pressure vanishes, but also the so-called dilaton charge vanishes [6]. The dilaton charge is the source for the dilaton equation of motion following from the action (2.1). Since the value of the dilaton charge is frame dependent we work in the string frame where the metric is $\hat{g}_{\mu\nu} = e^{\phi/2} g_{\mu\nu}$. Expanding the string frame metric around the asymptotically flat region $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$, we get from (3.4)

$$\begin{aligned}
\hat{h}_{00} &= \left[\alpha - \delta_1 - 2(7-p)\alpha\right] \frac{\omega^{6-p}}{r^{6-p}} \\
\hat{h}_{ij} &= (\delta_1 - \alpha)\frac{\omega^{6-p}}{r^{6-p}}\delta_{ij}\,, \qquad i,j = 1,\ldots,p+1 \\
\hat{h}_{mn} &= (\delta_1 - \alpha)\frac{\omega^{6-p}}{r^{6-p}}\delta_{mn}\,, \qquad m,n = p+2,\ldots,9 \\
\hat{h} &= \eta^{\mu\nu}\hat{h}_{\mu\nu} = 2\left[5\delta_1 - \alpha(p-2)\right]\frac{\omega^{6-p}}{r^{6-p}}\,.
\end{aligned} \tag{3.8}$$

The linearized equation of motion of the dilaton in the string frame takes the form

$$\nabla^2\left(\hat{h}_{mm} - \hat{h} + 4\phi\right) = -2\kappa_0^2 Q_D \delta^{(8-p)}(r)\,. \tag{3.9}$$

Whence we obtain,

$$Q_D = \frac{\Omega_{7-p}}{2\kappa_0^2}(6-p)(\alpha - \delta_1)\omega^{6-p}\,, \tag{3.10}$$

where ϕ was calculated from (3.4) as $\phi = (2\delta_1 - \frac{p-3}{2}\alpha)\frac{\omega^{6-p}}{r^{6-p}}$. Thus we find that for the dilaton charge to vanish

$$\alpha = \delta_1 \tag{3.11}$$

Using (3.7) and (3.11) we obtain from (2.18) in $d = 10$

$$\alpha = -\sqrt{\frac{4}{(6-p)(7-p)}} = \delta_1 = \frac{2\delta_2}{6-p}\,. \tag{3.12}$$

Substituting (3.12) into (3.4) we find the supergravity configuration of tachyon matter as,

$$ds^2 = \left(\frac{H}{\tilde{H}}\right)^{\frac{1}{4}\sqrt{\frac{7-p}{6-p}}} \left[-\left(\frac{H}{\tilde{H}}\right)^{-2\sqrt{\frac{7-p}{6-p}}} dt^2 + \sum_{i=1}^{p+1} dx_i^2 + (H\tilde{H})^{\frac{2}{6-p}} \left(dr^2 + r^2 d\Omega_{7-p}^2\right)\right]$$

$$e^{2\phi} = \left(\frac{H}{\tilde{H}}\right)^{-\sqrt{\frac{7-p}{6-p}}}, \qquad F_{[8-p]} = 0\,. \tag{3.13}$$

This is precisely the tachyon matter configuration obtained in [6] using a different method. We have thus seen how the Wick rotated solutions or the temporally delocalized non-supersymmetric p-branes (3.4) can be regarded as the interpolating solutions between non-BPS D$(p+1)$-branes and the tachyon matter, similar to the picture of rolling tachyon [16] on the non-BPS D-branes discussed by Sen. We like to point out that although the rolling tachyon implies that the tachyon is time dependent, the supergravity solutions are still static. The reason is, the supergravity configurations represent S-branes delocalized in the time direction and unless the time direction is fully localized the supergravity configurations will remain static. In the approach of [6], the question of time independence of the supergravity configurations or even why one should start with the non-supersymmetric black p-brane solutions [18] to arrive at tachyon matter was not clear. Our approach, however, clarifies these points.

4. Conclusion

To summarize, in this paper we have constructed non-supersymmetric spatially delocalized (in one transverse direction) p-brane solutions of type II supergravities in d space-time dimensions. Unlike the localized solutions (which contain three parameters), the delocalized solutions are characterized by four parameters. We have shown how these solutions in $d = 10$ nicely interpolate between non-BPS D$(p+1)$-branes (of the type discussed by Sen) and the BPS Dp-branes. This process is very similar to the picture of tachyon condensation on the tachyonic kink solution of the non-BPS D$(p+1)$-branes. In our approach we have clarified the reasons for the appearance of even and odd dimensional non-BPS D-branes in type IIB and type IIA string theories respectively. Further, we have obtained non-supersymmetric, temporally delocalized Euclidean p-brane solutions by an Wick rotation on our previous solutions. We have shown how these latter solutions nicely interpolate between non-BPS D$(p+1)$-branes and the tachyon matter supergravity configurations. This process is very similar to the picture of rolling tachyon on the non-BPS D$(p+1)$-branes. Our approach also clarifies why we need a static solution to understand the tachyon condensation for the time-dependent tachyon or the rolling tachyon on the non-BPS D-branes. We emphasize that although we have indicated the similarities of our approach with the process of tachyon condensation for the space dependent as well as the time dependent tachyons, it would be nice to understand the physical meanings of the parameters and the exact relationships of them with the dynamics of tachyon condensation.

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